# Quantum Mathematics

Ella Simmons
Fall 2022 Semester

## Overview

Independent Study Advisor: Dr. Prudhom

Quantum physics is at the bleeding edge of science: its laws will dictate the next century of scientific and technological development, and at the core of these concepts is math, which ultimately forms the fabric of the universe. Quantum math covers a wide variety of topics, typically taught independently, including linear algebra, complex number, and a specific form of notion used in the quantum fields. Learned together, with the added context of quantum topics, their interconnections and relevance will be far more apparent, and a solid foundation for any future pursuit of the quantum fields will be developed.

## Preparation

*STAQ Summer School and QuibitxQuibit Camp*

The summer between tenth and eleventh grade, I attended two summer camps focused on quantum physics and quantum computing: the STAQ Summer School and the QuibitxQuibit High School Camp. The QuibitxQuibit camp is a virtual, two week summer course open to high schoolers with an interest in quantum computing and geometry under their belt. STAQ Summer School is a Duke based "crash course" in quantum, taught through both videos and virtual lectures from quantum professors around the country, and typically open to only undergraduate, graduate, and postdoc students. Both gave me a solid baseline and foundation with the basics of quantum theory, and kindled a love of the topic.

*Quantum and Optics Lab*

The summer between eleventh and twelfth grade (this current summer), I am working in a paid intern position at Dr. Ken Brown's labs at Duke, which are focused around quantum computing and optics. There are two major current projects in experimental stages at the labs: using "laser-cooled atomic ions to sympathetically cool, control, and probe molecular ions" and improving current quantum computers through "developing better hardware, discovering control sequences that reduce errors, and designing quantum error correction codes". Both of these projects utilize the foundations of quantum physics and quantum theory.

## Process

### *Main External Materials*

Griffiths, David J. *Introduction to Quantum Mechanics*. Reed College, 23 Nov. 2020, https://www.academia.edu/44562863/Mecanica_Quantica_David_Griffiths.

Laforest, Martin. *The Mathematics of Quantum Mechanics*. University of Waterloo and the Institute for Quantum Computing, http://www.stat.ucla.edu/~ywu/linear.pdf.

Nielsen, Michael A., and Isaac L. Chuang. *Quantum Computation and Quantum Information*. Cambridge University Press, http://mmrc.amss.cas.cn/tlb/201702/W020170224608149940643.pdf.

Woit, Peter. *Quantum Theory, Groups and Representations: An ... - Columbia University*. Department of Mathematics, Columbia University, https://www.math.columbia.edu/~woit/QMbook/qmbook-latest.pdf.

*Meetings*

Meetings with Dr. Pruhdom will occur every Day 3 during lunch in Dr. Prudhom's room

*Preliminary Schedule*

I. **Unit One:** Linear Algebra and Complex Numbers Crash Course
   A. Introductions to the textbooks
      1. Read preface of The Mathematics of Quantum Mechanics
      2. Read preface of Quantum Theory, Groups, and Representations
   B. Complex Numbers
      1. Read and take notes on Chapter One: Complex Numbers in The Mathematics of Quantum Mechanics
      2. Read Why are complex numbers needed in quantum mechanics?
      3. Watch Necessity of complex numbers from MIT 8.04 Quantum Physics 1
      4. Read Section 1: Complex Numbers in Quantum Mechanics from Illinois Course Phys 580
      5. Compile notes and information into write-up
   C. Linear Algebra Basics
      1. Read and take notes on Chapter Two: Linear Algebra in The Mathematics of Quantum Mechanics
      2. Read through Lecture 8: A Crash Course in Linear Algebra from UCSB Math/CS 120
      3. Watch video in the Matrix Transformations of Khan Academy's Linear Algebra Course
      4. Read and take notes on Chapter Four: Linear Algebra Review, Unitary and Orthogonal Groups in Quantum Theory, Groups, and Representations
      5. Compile notes and information into write-up
   D. Products
      1. Compile both write-ups into section one of final product (textbook)
      2. Create educational mini-lesson video on Linear Algebra
      3. Create education mini-lesson video on Complex Numbers
II. **Unit Two**: Quantum Mechanics Overview
   A. Basic Principles of Quantum Mechanics

1. Read 1.1: Introduction in Quantum Theory, Groups, and Representations
2. Read and take notes on 1.2 : Basic Principles of Quantum Mechanics in Quantum Theory, Groups, and Representations
3. Read and take notes on Chapter 3: Quantum Mechanics in The Mathematics of Quantum Mechanics
4. Read and take notes on 2.2: The Postulates of Quantum Mechanics in Quantum Computation and Quantum Information
5. Compile notes and information into write-up

B. Representation in Quantum Mechanics
   1. Read and take notes on the rest of Chapter One: Introduction and Overview (1.3-1.6) in Quantum Theory, Groups, and Representations
   2. Read and take notes on Chapter Two: The Group U(1) and its Representations in Quantum Theory, Groups, and Representations
   3. Read and take notes on Chapter Three: Two-state Systems and SU(2)
   4. Compile notes and information into write-up

C. The Wave Function and Schrodinger's Equation
   1. Read and take notes on Chapter One: The Wave Function in Introduction to Quantum Mechanics
   2. Read and take notes on Chapter Two: The Time-Independent Schrodinger Equation in Introduction to Quantum Mechanics
   3. Compile notes and information into write-up

D. Products
   1. Compile all three write-ups into section one of final product (textbook)
   2. Create educational mini-lesson video on the Basic Principles of Quantum Mechanics
   3. Create education mini-lesson video on Representation in Quantum Mechanics
   4. Create education mini-lesson video on The Wave Function and Schrodinger's Equation

III. **Unit Three**: Quantum-Specific Linear Algebra, Lie Algebras and the Basics of Quantum Computing

A. Quantum-Specific Linear Algebra
   1. Read 8.1: Linear Algebra in Learn Quantum Computation using Qiskit
   2. Read Linear Algebra for Quantum Computation
   3. Read 3.1: Linear Algebra in Introduction to Quantum Mechanics
   4. Read and take notes on 2.1: Linear Algebra in Quantum Computation and Quantum Information
   5. Read and take notes on Chapter Four: Linear Algebra Review in Quantum Theory, Groups, and Representations
   6. Compile notes and information into write-up

B. Lie Algebras
   1. Watch What is a Lie Algebra?
   2. Read Introduction to Lie Algebras

3. Read and take notes on Chapter 5: Lie Algebras and Lie Algebra Representations in Quantum Theory, Groups, and Representations
4. Compile notes and information into write-up

C. Basics of Quantum Computing
1. Read Chapter One: Quantum States and Qubits in Learn Quantum Computation using Qiskit
2. Read Chapter Two: Multiple Qubits and Entanglement in Learn Quantum Computation using Qiskit
3. Read Quantum Protocols and Quantum Algorithms in Learn Quantum Computation using Qiskit
4. Read and take notes on Chapter One: Introduction and Overview in Quantum Computation and Quantum Information
5. Read and take notes Chapter Three: Introduction to Computer Science in Quantum Computation and Quantum Information
6. Compile notes and information into write-up

D. Products
1. Compile all three write-ups into section one of final product (textbook)
2. Create educational mini-lesson video on Quantum-Specific Linear Algebra
3. Create education mini-lesson video on Lie Algebras
4. Create education mini-lesson video on The Basics of Quantum Computing

IV. **Unit Four**: Rotation, Spin Groups, Tensor Products and Momentum

A. Rotation and Spin Groups
1. Watch What is Quantum Spin and What is Spin? | Quantum Mechanics
2. Read and take notes on Chapter Six: The Rotation and Spin Groups in 3 and 4 Dimensions in Quantum Theory, Groups, and Representations
3. Read and take notes on Chapter Sever: Rotations and the Spin ½ Particle in a Magnetic Field in Quantum Theory, Groups, and Representations
4. Look through Quantum Mechanical Spin lecture
5. Compile notes and information into write-up

B. Representations of SU(2) and SO(3)
1. Read The Irreducible Representations of the Lie algebra SU(2)
2. Read and take notes on The Groups SO(3) and SU(2) and Their Representations
3. Read and take notes on Chapter Eight: Representations of SU(2) and SO(3) in Quantum Theory, Groups, and Representations
4. Compile notes and information into write-up

C. Tensor Products and Entanglement
1. Read and take notes on The Tensor Product, Demystified
2. Watch A Concrete Introduction to Tensor Products
3. Read and take notes on Chapter Nine: Tensor Products, Entanglement and Additional Spin in Quantum Theory, Groups, and Representations
4. Compile notes and information into write-up

D. Momentum and the Free Particle
    1. Watch The Wave for A Free Particle and The Free Quantum Particle
    2. Read 5.1: The Free Particle from LibreTexts Chemistry and 3.4: The Quantum Mechanical Free Particle from LibreTexts Chemistry
    3. Read and take notes on Chapter Ten: Momentum and the Free Particle in Quantum Theory, Groups, and Representations
    4. Compile notes and information into write-up
E. Products
    1. Compile all four write-ups into section one of final product (textbook)
    2. Create educational mini-lesson video on Rotation and Spin Groups
    3. Create education mini-lesson video on Representations of SU(2) and SO(3)
    4. Create education mini-lesson video on Tensor Products and Entanglement
    5. Create education mini-lesson video on Momentum and the Free Particle

V. **Unit Five**: Quantum Computation
A. Quantum Circuits
    1. Read 3.1: Defining Quantum Circuits in Learn Quantum Computation using Qiskit
    2. Watch Qubits and Quantum States, Quantum Circuits, Measurements Part One and Part Two
    3. Read and take notes on Chapter Four: Quantum Circuits in Quantum Computation and Quantum Information
    4. Read Making quantum circuits more robust from MIT News
    5. Compile notes and information into write-up
B. Fourier Analysis and the Free Particle
    1. Read 3.5: Quantum Fourier Transform in Learn Quantum Computation using Qiskit
    2. Watch But what is the Fourier Transform? A visual introduction.
    3. Read and take notes on Chapter Eleven: Fourier Analysis and the Free Particle in Quantum Theory, Groups, and Representations
    4. Read and take notes on Chapter Five: The Quantum Fourier Transform and Its Applications in Quantum Computation and Quantum Information
    5. Compile notes and information into write-up
C. Quantum Algorithms
    1. Read Chapter Three: Quantum Protocols and Quantum Algorithms in Learn Quantum Computation using Qiskit
    2. Read Chapter Four: Quantum Algorithms for Application in Learn Quantum Computation using Qiskit
    3. Read and take notes on Chapter Six: Quantum Search Algorithms in Quantum Computation and Quantum Information
    4. Experiment with code in Quantum Inspire Knowledge Base
    5. Read Quantum Algorithms and Applications
    6. Explore the Quantum Algorithm Zoo

    7. Compile notes and information into write-up

D. Quantum Computers
    1. Read Chapter Five: Investigating Quantum Hardware Using Quantum Circuits in Learn Quantum Computation using Qiskit
    2. Read and take notes on Chapter Seven: Quantum Computers: Physical Realization in Quantum Computation and Quantum Information
    3. Watch Quantum Computers, Explained With Quantum Physics, Quantum Computers Explained - Limits of Human Technology, and How Does a Quantum Computer Work
    4. Explore news stories about latest quantum computer advancements
    5. Compile notes and information into write-up

E. Products
    1. Compile all four write-ups into section one of final product (textbook)
    2. Create educational mini-lesson video on Quantum Circuits
    3. Create education mini-lesson video on Fourier Analysis and the Free Particle
    4. Create education mini-lesson video on Quantum Search Algorithm
    5. Create education mini-lesson video on Quantum Computers

Each unit will last roughly a month.
Unit 1 goes from August 29 - September 23.
Unit 2 goes from September 23 - October 29.
Unit 3 goes from October 29 - November 29.
Unit 4 goes from November 29 to January 1
Unit 5 goes from January 1 to the end of the semester.
Therefore, each subsection lasts about a week.

## Final Product & Documentation

*Overarching Goal*

Ultimately, I want to be able to express what I have learned in an accessible and approachable way. I believe that it is much more difficult, but also rewarding, to not just learn something for yourself, but be able to express it to others. Thus, through my final product and documentation, I want to communicate what I have learned to those who do not have a background in the topic. I want to communicate it to people, who like me a couple years ago, couldn't break into a concept armed only with curiosity.

*Documentation*

In accordance with the overarching goal, documentation would include mini-lessons on each unit covered. These mini-lessons would be recorded and serve as a fun, informative, video lecture, in the range of ten to twenty minutes in length. Documenting this way practices communication and representation skills and requires me to fully understand the material, all of which will be good practice for the final product.

*Final Product*
To culminate this independent project, I want to create a beginners' textbook and guide to understand quantum mathematics, and the basics of quantum mathematics. Each of my studying units would form a chapter in the textbook, which would be aimed at those with no background or previous knowledge of the subject. A final product like this forces me to not only understand the concepts enough to teach them and practice my writing communication skills (one of the most important skills in STEM fields!), but also requires me to process the information and find an easier and more accessible way to present it.

## Major Commitments

*Schedule*
AP Biology, Senior English Electives, Discrete Mathematics, Introduction to Topography, Acting Studio, Linear Algebra, The Science in Science Fiction, Planetary Science

*Extracurriculars*
DARC SIDE
Technical Theater

## Signatures

This proposal has been submitted sincerely and with adequate planning. The independent study is manageable in scope (both within the semester time limit and the academic schedule of the student).

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| Print: | Ella Simmons | Andrew Prudhom | Paul Slack | Rebecca Simmons |
| | Student | IS Advisor | HR Advisor | Parent |
| Sign: | | | | |